Exercises 2.2

Most of the following equations are identities. Some may be conditional equations. Determine in which category each belongs and prove your conclusion. Give the replacement set for each equation.

1. $\sec x \sin x \equiv \tan x$.
2. $\sec \phi \cot \phi \equiv \csc \phi$.
3. $\tan y \csc y \equiv \sec y$.
4. $\csc \theta \cos \theta \equiv \cot \theta$.
5. $\sin \alpha \cot \alpha \equiv \cos \alpha$.
6. $\cos \beta \tan \beta \equiv \sin \beta$.
7. $\sec^2 x - \tan^2 x \equiv 1$.
8. $\csc^2 \beta - \cot^2 \beta \equiv 1$.
9. $\csc^2 y - \sec^2 y \equiv \sin^2 y$.
10. $\cos(-\theta) \cos \theta - \sin \theta \sin(-\theta) \equiv 1$.
11. $\tan^2 \theta \csc^2 \theta - \sec^2 \theta \sin^2 \theta \equiv 1$.
12. $\cos^2 y \tan^2 y + \sin^2 y \cot^2 y \equiv 1$.
13. $\sec^2 \phi \cot^2 \phi - \csc^2 \phi \cos^2 \phi \equiv 1$.
14. $\cos^4 \alpha - \sin^4 \alpha \equiv \cos^2 \alpha - \sin^2 \alpha$.
15. $\sec^4 x - \tan^4 x \equiv \sec^2 x + \tan^2 x$.
16. $\sec^4 \phi - 1 \equiv \tan^2 \phi (\sec^2 \phi + 1)$.
17. $\csc^4 z - \cot^4 z \equiv \csc^2 z + \cot^2 z$.
18. $\tan^3 \alpha + 1 \equiv (\tan \alpha + 1)(\sec^2 \alpha - \tan \alpha)$.
19. $\tan^3 \alpha - 1 \equiv (\tan \alpha - 1)(\tan \alpha + \sec^2 \alpha)$.
20. $\cot^3 \beta + 1 \equiv (\cot \beta + 1)(\csc^2 \beta - \cot \beta)$.
21. $\cot^3 \beta - 1 \equiv (\cot \beta - 1)(\cot \beta + \csc^2 \beta)$.
22. $\cos^6 \gamma - \sin^6 \gamma \equiv \cos^2 \gamma + \cos^4 \gamma$.
23. $\cos^6 \phi - \sin^6 \phi \equiv \cos^4 \phi + \sin^2 \phi$.
24. $\cos^3 x + \sin^3 x \equiv \cos x(1 - \cos x \sin x) + \sin x(1 - \cos x \sin x)$.
25. $\cos^3 y + \sin^3 y \equiv \cos y(1 - \cos y \sin y) - \sin y(\cos y \sin y - 1)$.
26. $\cos^3 \alpha - \sin^3 \alpha \equiv \cos \alpha(1 + \sin \alpha \cos \alpha) - \sin \alpha(1 + \sin \alpha \cos \alpha)$.
27. $(\tan^2 \gamma + 1)^2 \equiv (-\sec \gamma)^4$.
28. $(\cos x - \sin x)^2 \equiv 1 - 2 \sin x \cos x$.
29. $(\tan \alpha - 1)^2 \equiv \sec^2 \alpha - 2 \tan \alpha$.
30. $(1 + \cot z)^2 \equiv 2 \cot z + \csc^2 z$.
31. $(\cos \beta + \sin \beta)^4 \equiv \cos^2 \beta + \sin^2 \beta + 4 \cos^2 \beta \sin^2 \beta + 4 \cos \beta \sin \beta$.
32. $(\cos \alpha - \sin \alpha)^4 \equiv \cos^2 \alpha + \sin^2 \alpha + 4 \cos^2 \alpha \sin^2 \alpha - 4 \cos \alpha \sin \alpha$.
33. $(\sin \theta + \cos \theta)^3 \equiv \sin \theta + 2 \sin \theta \cos \theta (\sin \theta + \cos \theta) + \cos \theta$.
34. $\cos^2 x - \sin^2 x \equiv 1 - 2 \sin^2 x$.
35. $\cos^2 \alpha - \sin^2 \alpha \equiv 2 \cos^2 \alpha - 1$.
36. $\cos^4 \theta \equiv 1 - 2 \sin^2 \theta + \sin^4 \theta$.
37. $\sec^2 \phi + \csc^2 \phi \equiv \sec^2 \phi \csc^2 \phi$.
38. $\sec^4 x - \tan^4 x \equiv \dfrac{\sin^2 x + 1}{\cos^2 x}$.
39. $\csc^4 \beta - \cot^4 \beta \equiv \dfrac{\cos^2 \beta + 1}{\sin^2 \beta}$.
40. $\csc^4 \alpha - 1 \equiv \cot^2 \alpha (\csc^2 \alpha + 1)$.
41. $\tan^2 \gamma - \cot^2 \gamma \equiv \dfrac{\sin^2 \gamma - \cos^2 \gamma}{\cos^2 \gamma \sin^2 \gamma}$.
42. $\tan^2 z + \cot^2 z \equiv \dfrac{2 \sin^4 z - 2 \sin^2 z + 1}{\sin^2 z \cos^2 z}$.
43. $\tan^2 \phi + \cot^2 \phi \equiv 2 \tan^2 \phi - 2 \sec^2 \phi + \sec^2 \phi \csc^2 \phi$.
44. $\dfrac{\tan y + \cot y}{\sec y \csc y} \equiv 1$.
45. $\dfrac{\sin \phi + \cos^2 \phi \csc \phi}{\csc \phi} \equiv 1$.